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Unification of Frequency Direction PACE Algorithms for OFDM

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Abstract—Frequency direction Pilot-symbol Aided Channel Estimation (PACE) for Orthogonal Frequency Division Multiplexing (OFDM) is crucial in high-rate wireless systems. The choice of an estimator for upcoming standards, such as the Long Term Evolution (LTE) of UTRA, has to take into account their specificities, namely the presence of virtual subcarriers and non-sample-spaced channels. To ease this choice, we propose a unified presentation of estimators encompassing most of the algorithms that can be found in literature, which only differ by the assumptions made on the channel. This unification leads to common Mean Squared Error (MSE) expression, both for sample-spaced and non sample-spaced channels, and enables easy, yet comprehensive comparisons between the estimators.

I. INTRODUCTION

In the frame of OFDM for upcoming wireless systems, much attention has been given to pilot-based channel estimators (PACE) showing that the performance tradeoff of the algorithms depends on the relationship between the Power Delay Profile (PDP) properties and the frequency-domain pilot spacing. Deterministic approaches have, so far, been separated into time- and frequency-domain solutions. Deterministic time-domain solutions are: the Time-Domain Least Squares (TDLS) [3], [4], the Maximum Likelihood (ML) approach [5], [6] and the Noise Reduction Algorithm (NRA) [7]. Deterministic frequency-domain methods are Spline, Gaussian or Lagrange interpolation, and require higher pilot overhead to achieve an acceptable performance [8]. Bayesian approaches such as the Minimum Mean Squared Error (MMSE) estimator in time domain and/or frequency domain have been proposed in [2], [3], with complexity reduction by Singular Value Decomposition (SVD) suggested in [9].

The major contribution of this paper is to provide a framework for the choice of a channel estimation algorithm for the upcoming PACE OFDM-based standards. In this study we derive a uniform algorithm and Mean Squared Error (MSE) formulation, covering all studied algorithms and thereby facilitating a generic performance comparison. Three main effects will be studied: the impact of a priori knowledge in a full bandwidth system with a SS channel, the effect of virtual subcarriers and the effect of a NSS channel. Performance simulations are conducted in a LTE context and will show the importance of knowing the exact tap delays, for the studied algorithms, at the receiver in order to avoid the leakage effect due to NSS channel.

II. ANALYTICAL MODEL

A. Multipath Channel Model

The OFDM signal is transmitted over a block fading normalized multipath Rayleigh channel with a Channel Impulse Response (CIR) given by:

$$g(\tau) = \sum_{i=0}^{N-1} a_i \delta(\tau - \tau_i) \text{ with } \sum_{i=0}^{N-1} E\{|a_i|^2\} = 1 \quad (1)$$

where a_i are the different wide sense stationary, uncorrelated complex Gaussian random path gains with their corresponding time delays τ_i , N is the number of paths and τ_{N-1} is assumed to be smaller than the cyclic prefix.

B. Baseband Model

Due to spectral constraints, many multicarrier systems make use of only a subset of $N_u < N_{fft}$ subcarriers, leaving unused the $N_{fft} - N_u$ remaining ones, usually placed at the edges of the transmission bandwidth. The latter are the so-called virtual subcarriers, and this scenario will be referred to as Partial Bandwidth, where N_{fft} is the FFT size. In such a context, the received signal at the used subcarriers can be described by:

$$\mathbf{y}_u = \mathbf{D}_u \mathbf{h}_u + \mathbf{w}_u = \mathbf{D}_u \mathbf{F}_u \mathbf{g} + \mathbf{w}_u \quad (2)$$

where the (frequency) Channel Transfer Function (CTF) at the used subcarrier positions $\mathbf{h}_u \in \mathbb{C}^{N_u}$ is:

$$\mathbf{h}_u = \mathbf{F}_u \mathbf{g} \quad (3)$$

$\mathbf{D}_u \in \mathbb{C}^{N_u \times N_u}$ is a diagonal matrix with the transmitted symbols at the used subcarriers, $\mathbf{w}_u \in \mathbb{C}^{N_u}$ is the AWGN vector corresponding to the used subcarriers, and $\mathbf{F}_u \in \mathbb{C}^{N_u \times N_{fft}}$ is a subset of the Fourier matrix \mathbf{F} with $\mathbf{F}_u[k, n] = \mathbf{F}[k, n] = e^{-j2\pi \frac{nk}{N_{fft}}}$ for $\frac{-N_u}{2} \leq k \leq \frac{N_u}{2} - 1$.

C. Received Signal at Pilot Subcarriers

N_p pilot symbols are transmitted in positions $\{p_m, 0 \leq m \leq N_p - 1\}$. The received signal in these pilot subcarriers can be then written as:

$$\mathbf{y}_p = \mathbf{D}_p \mathbf{h}_p + \mathbf{w}_p = \mathbf{D}_p \mathbf{F}_p \mathbf{g} + \mathbf{w}_p \quad (4)$$

$\mathbf{D}_p \in \mathbb{C}^{N_p \times N_p}$, $\mathbf{h}_p \in \mathbb{C}^{N_p}$, $\mathbf{F}_p \in \mathbb{C}^{N_p \times N_{fft}}$ and $\mathbf{w}_p \in \mathbb{C}^{N_p}$ with $\mathbf{D}_p[m, m] = \mathbf{D}_u[p_m, p_m]$, $\mathbf{h}_p[m] = \mathbf{h}_u[p_m]$, $\mathbf{F}_p[m, n] = \mathbf{F}_u[p_m, n]$ and $\mathbf{w}_p[m] = \mathbf{w}_u[p_m]$.

III. CHANNEL ESTIMATION ALGORITHMS

The initial Least Squares (LS) estimate at the pilots is :

$$\mathbf{h}_{ls} = \mathbf{D}_p^{-1} \mathbf{y}_p = \mathbf{h}_p + \mathbf{D}_p^{-1} \mathbf{w}_p \quad (5)$$

The pilot symbols are M-PSK modulated with unit power and the number of pilot symbols used N_p is assumed to be larger than the normalized maximum delay of the channel. In the following, at sampling rate τ_s , two scenarios are considered:

Case 1 A SS-CIR scenario, where it is assumed that the delays τ_i are sample spaced on the same grid as the receiver and all $\frac{\tau_i}{\tau_s}$ are integer values.

Case 2 A NSS-CIR scenario, where it is assumed that the delays τ_i are not sample spaced on the same grid as the receiver and some $\frac{\tau_i}{\tau_s}$ are not integer values.

A. Sample-Spaced Channel

The different studied algorithms can be written in the following generic formula:

$$\mathbf{h}_{est} = \mathbf{F}_{ux} \mathbf{g}_{est} = \mathbf{F}_{ux} \mathbf{M}_{est} \mathbf{h}_{ls} \quad (6)$$

which will be specified for each estimator.

1) *Time-Domain Least Squares*: This estimator [3], [4] assumes no a priori knowledge of the channel, and estimates $N_x = N_p$ samples of \mathbf{g} , corresponding to $\mathbf{g}_p[n] = \mathbf{g}[n]$ for $0 \leq n \leq N_p - 1$. The formulation of TDLS is:

$$\mathbf{h}_{tdls} = \mathbf{F}_{up} \mathbf{g}_{tdls} = \mathbf{F}_{up} \mathbf{F}_{pp}^{-1} \mathbf{h}_{ls} \quad (7)$$

where $\mathbf{F}_{pp} \in \mathbb{C}^{N_p \times N_p}$ and $\mathbf{F}_{up} \in \mathbb{C}^{N_u \times N_p}$ correspond, respectively, to $\mathbf{F}_{pp}[m, n] = \mathbf{F}_p[m, n]$ and $\mathbf{F}_{up}[k, n] = \mathbf{F}_u[k, n]$ for $0 \leq n \leq N_p - 1$. For the TDLS estimator, then, $\mathbf{M}_{est} = \mathbf{F}_{pp}^{-1}$. Note that \mathbf{F}_{pp} is always invertible due to the Vandermonde structure of the DFT matrix [4]; however, in a Partial Bandwidth scenario, this matrix can become ill-conditioned depending on the number of virtual subcarriers.

2) *Maximum Likelihood*: The ML estimator [5], [6], assumes that the receiver knows the CIR length, i.e., the last channel path's delay τ_{N-1} , and only estimates the $N_x = N_s = \frac{\tau_{N-1}}{\tau_s} + 1$ first samples of the SS-CIR, corresponding to $\mathbf{g}_s[n] = \mathbf{g}[n]$ for $0 \leq n \leq N_s - 1$. The ML estimator is expressed as:

$$\mathbf{h}_{ml} = \mathbf{F}_{us} \mathbf{g}_{ml} = \mathbf{F}_{us} (\mathbf{F}_{ps}^H \mathbf{F}_{ps})^{-1} \mathbf{F}_{ps}^H \mathbf{h}_{ls} \quad (8)$$

where $\mathbf{F}_{ps} \in \mathbb{C}^{N_p \times N_s}$ and $\mathbf{F}_{us} \in \mathbb{C}^{N_u \times N_s}$ correspond, respectively, to $\mathbf{F}_{ps}[m, n] = \mathbf{F}_p[m, n]$ and $\mathbf{F}_{us}[k, n] = \mathbf{F}_u[k, n]$ for $0 \leq n \leq N_s - 1$. In this case, $\mathbf{M}_{est} = (\mathbf{F}_{ps}^H \mathbf{F}_{ps})^{-1} \mathbf{F}_{ps}^H$. Similarly to the case of the TDLS estimator, the matrix \mathbf{F}_{ps} is always of full column rank (for $N_p \geq N_s$), implying that $\mathbf{F}_{ps}^H \mathbf{F}_{ps}$ is of full rank. However, in the presence of virtual subcarriers this matrix can become ill-conditioned, as for the TDLS estimator.

3) *Noise Reduction Algorithm* : As a solution to the ill-conditioning problems [11] of the previous estimators, a small value can be added to the diagonal of the matrix to be inverted [7], thus avoiding numerical instability :

$$\mathbf{h}_{nra} = \mathbf{F}_{us} \mathbf{g}_{nra} = \mathbf{F}_{us} (\mathbf{F}_{ps}^H \mathbf{F}_{ps} + \gamma_{nra} \mathbf{I}_s)^{-1} \mathbf{F}_{ps}^H \mathbf{h}_{ls} \quad (9)$$

where \mathbf{I}_s is the identity matrix of size N_s , and γ_{nra} is a positive scalar value. From (9), it follows that $\mathbf{M}_{est} = (\mathbf{F}_{ps}^H \mathbf{F}_{ps} + \gamma_{nra} \mathbf{I}_s)^{-1} \mathbf{F}_{ps}^H$. In a Full Bandwidth scenario with evenly spaced pilot subcarriers, it can be shown that the optimum value is $\gamma_{nra} = N_s \sigma_w^2$.

4) *Enhanced Noise Reduction Algorithm* : The Enhanced Noise Reduction Algorithm (ENRA) differs from the NRA by only estimating the $N_x = N_t$ samples of \mathbf{g} which are not null, i.e., $\mathbf{g}_t[n] = \mathbf{g}[\tau_n/\tau_s]$ for $0 \leq n \leq N_t - 1$. Therefore, the knowledge of the number of paths and their corresponding delays is required. The estimator is given by:

$$\mathbf{h}_{enra} = \mathbf{F}_{ut} \mathbf{g}_{enra} = \mathbf{F}_{ut} (\mathbf{F}_{pt}^H \mathbf{F}_{pt} + \gamma_{enra} \mathbf{I}_t)^{-1} \mathbf{F}_{pt}^H \mathbf{h}_{ls} \quad (10)$$

where $\mathbf{F}_{pt} \in \mathbb{C}^{N_p \times N_t}$ and $\mathbf{F}_{us} \in \mathbb{C}^{N_u \times N_t}$ correspond, respectively, to $\mathbf{F}_{pt}[m, n] = \mathbf{F}_p[m, \tau_n/\tau_s]$ and $\mathbf{F}_{ut}[k, n] = \mathbf{F}_u[k, \tau_n/\tau_s]$ for $0 \leq n \leq N_t - 1$. \mathbf{I}_t denotes the identity matrix of size N_t . Hence, for the ENRA $\mathbf{M}_{est} = (\mathbf{F}_{pt}^H \mathbf{F}_{pt} + \gamma_{enra} \mathbf{I}_t)^{-1} \mathbf{F}_{pt}^H$. Analogously to the NRA, the value $\gamma_{enra} = N_t \sigma_w^2$ is optimum in a Full Bandwidth with equally spaced pilots scenario.

5) *Wiener Filter*: The Wiener filter (WF) estimator minimizes the MSE of the estimate by making use of channel and noise correlation properties, and has been broadly treated in literature [2], [9], [10], [12]. It is classically formulated as:

$$\mathbf{h}_{wf} = \mathbf{R}_{\mathbf{h}_u \mathbf{h}_p} (\mathbf{R}_{\mathbf{h}_p \mathbf{h}_p} + \sigma_w^2 \mathbf{I}_p)^{-1} \mathbf{h}_{ls} \quad (11)$$

where $\mathbf{R}_{\mathbf{h}_u \mathbf{h}_p} = \mathbf{E}\{\mathbf{h}_u \mathbf{h}_p^H\}$ is the correlation matrix of \mathbf{h}_u and \mathbf{h}_p , $\mathbf{R}_{\mathbf{h}_p \mathbf{h}_p} = \mathbf{E}\{\mathbf{h}_p \mathbf{h}_p^H\}$ is the autocorrelation matrix of \mathbf{h}_p , and \mathbf{I}_p is the identity matrix of size N_p . In the sample spaced case, it leads to:

$$\mathbf{h}_{wf} = \mathbf{F}_{ut} \mathbf{g}_{wf} = \mathbf{F}_{ut} (\mathbf{F}_{pt}^H \mathbf{F}_{pt} + \sigma_w^2 \mathbf{R}_{\mathbf{g}_t \mathbf{g}_t}^{-1})^{-1} \mathbf{F}_{pt}^H \mathbf{h}_{ls} \quad (12)$$

For WF, $\mathbf{M}_{est} = (\mathbf{F}_{pt}^H \mathbf{F}_{pt} + \sigma_w^2 \mathbf{R}_{\mathbf{g}_t \mathbf{g}_t}^{-1})^{-1} \mathbf{F}_{pt}^H$.

Note that when no information about the channel correlation is available, a robust design of the filter is proposed, and consists of assuming a sample-spaced PDP with N_s samples and equal mean power in all taps. In such conditions $\mathbf{F}_{ut} = \mathbf{F}_{us}$, $\mathbf{F}_{pt} = \mathbf{F}_{ps}$ and $\mathbf{R}_{\mathbf{g}_t \mathbf{g}_t} = \frac{1}{N_s} \mathbf{I}_s$, showing that this robust WF implementation is equivalent to the NRA given in (9).

6) *Generic Formulation*: When observing the expressions of the studied algorithms, a general formulation that covers all the cases can be given by:

$$\mathbf{h}_{est} = \mathbf{F}_{ux} \mathbf{g}_{est} = \mathbf{F}_{ux} (\mathbf{F}_{px}^H \mathbf{F}_{px} + \gamma_{est} \mathbf{C}_{est})^{-1} \mathbf{F}_{px}^H \mathbf{h}_{ls} \quad (13)$$

An overview over the specific values taken by each element of (13) is given in Table 1.

Table 1: Generalization of the algorithms

Estimator	\mathbf{F}_{ux}	\mathbf{F}_{px}	γ_{est}	\mathbf{C}_{est}	\mathbf{g}_x	N_{est}
TDLS	\mathbf{F}_{up}	\mathbf{F}_{pp}	0	0	\mathbf{g}_p	N_p
ML	\mathbf{F}_{us}	\mathbf{F}_{ps}	0	0	\mathbf{g}_s	N_s
NRA	\mathbf{F}_{us}	\mathbf{F}_{ps}	$N_s\sigma_w^2$	\mathbf{I}_s	\mathbf{g}_s	N_s
ENRA	\mathbf{F}_{ut}	\mathbf{F}_{pt}	$N_t\sigma_w^2$	\mathbf{I}_t	\mathbf{g}_t	N_t
WF	\mathbf{F}_{ut}	\mathbf{F}_{pt}	σ_w^2	$\mathbf{R}_{\mathbf{g}_t\mathbf{g}_t}^{-1}$	\mathbf{g}_t	N_t

7) *MSE of the Estimators*: The different studied estimators are all described by (6). It is then possible to evaluate their respective performance by using one single generic MSE expression. The MSE is calculated as:

$$\text{MSE}\{\mathbf{h}_{est}[k]\} = \mathbb{E}\{|\mathbf{h}_u[k] - \mathbf{h}_{est}[k]|^2\} \quad (14)$$

Using (6), the MSE for the k^{th} subcarrier becomes:

$$\text{MSE}\{\mathbf{h}_{est}[k]\} = \mathbf{Q}[k, k] \quad (15)$$

where

$$\begin{aligned} \mathbf{Q} = & \mathbf{F}_{ux}[(\mathbf{I} - \mathbf{M}_{est}\mathbf{F}_{px})\mathbf{R}_{\mathbf{g}_x\mathbf{g}_x}(\mathbf{I} - \mathbf{F}_{px}^H\mathbf{M}_{est}^H) \\ & + \sigma_w^2\mathbf{M}_{est}\mathbf{M}_{est}^H]\mathbf{F}_{ux}^H \end{aligned} \quad (16)$$

Note that $\mathbf{R}_{\mathbf{g}_x\mathbf{g}_x} = \mathbb{E}\{\mathbf{g}_x\mathbf{g}_x^H\}$ depends on the a priori assumptions made by each estimator. The average MSE of the estimator can consequently be defined as:

$$\overline{\text{MSE}}\{\mathbf{h}_{est}\} = \frac{1}{N_u} \text{tr}\{\mathbf{Q}\} \quad (17)$$

In a Full Bandwidth ($N_u = N_{fft}$) scenario with a constant pilot spacing $\Delta_p = \frac{N_{fft}}{N_p}$, the products between the DFT-based matrices become diagonal matrices, and it is easy to simplify (16). Under such conditions, the MSE of the estimate becomes independent on the subcarrier index k . For the estimators which do not assume any knowledge of the mean power of the paths (TDLS, ML, NRA and ENRA), the MSE reduces to the generic expression:

$$\overline{\text{MSE}}\{\mathbf{h}_{est,full}\} = \frac{\gamma_{est}^2 + N_x N_p \sigma_w^2}{(N_p + \gamma_{est})^2}. \quad (18)$$

B. Non-Sample-Spaced Channel

In an NSS scenario, there is at least one path of the channel with a delay τ_i which is not an integer multiple of the sampling period τ_s . In this situation, the i^{th} column of the leakage matrix \mathbf{L}^1 will have non zero values for every element, i.e., $\mathbf{L}[n, i] \neq 0 \forall n$. \mathbf{a} is the vector of size N_t containing only the channel taps. As a consequence, the complex gain of the i^{th} path will have a contribution on all the samples of the equivalent SS-CIR. Fig. 1 illustrates how NSS paths are mapped to the equivalent SS-CIR for a simple example where $N_{fft} = 64$ and the channel is $g(\tau) = 0.8\delta(\tau - 0.5\tau_s) + 0.5\delta(\tau - 3.5\tau_s) + 0.3\delta(\tau - 7.5\tau_s)$. As

¹The relationship between \mathbf{g} and \mathbf{a} can be found to be: $\mathbf{g} = \frac{1}{N_{fft}}\mathbf{F}^H\mathbf{T}\mathbf{a} = \mathbf{L}\mathbf{a}$ where $\mathbf{T}[k, i] = e^{-j2\pi\frac{k}{N_{fft}}\frac{\tau_i}{\tau_s}}$ and $\mathbf{L}[n, i] = \frac{1}{N_{fft}} \frac{\sin(\pi\frac{\tau_i}{\tau_s})}{\sin(\frac{\pi}{N_{fft}}(n - \frac{\tau_i}{\tau_s}))} e^{-j\frac{\pi}{N_{fft}}((N_{fft}-1)\frac{\tau_i}{\tau_s} + n)}$, $\mathbf{L} \in \mathbb{C}^{N_{fft} \times N}$ is the leakage matrix, and represents how the complex gain a_i of each channel path is mapped to the SS-CIR.

can be seen, most of the power of each path is mapped to the surrounding samples in the SS-CIR. It is especially interesting how the last samples have significant amplitude, due to the leakage of the first channel paths.

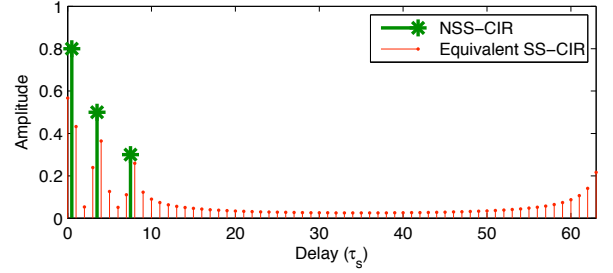


Fig. 1. Leakage of the NSS-CIR paths to the equivalent SS-CIR

The estimators studied in the sample-spaced case rely on the fact that most of the samples of \mathbf{g} are zero, and thus they can be canceled in the estimation problem. Obviously, this assumption does not hold any more in the NSS scenario, and the estimators must be modified accordingly. Due to the ill-condition problems of the TDLS and ML estimators, only NRA, ENRA and WF will be considered in the following.

1) *Modified NRA*: The NRA algorithm for SS channel is based on the knowledge of the CIR length, i.e., the maximum excess delay of the channel, so that every sample of \mathbf{g} beyond this value is assumed to be zero. For the NSS scenario, however, the length of the SS-CIR is N_{fft} due to the leakage effect, which will cause a performance degradation if $N_p < N_{fft}$. Since the actual path delays are considered unknown, the selection of the samples to estimate can only be approximated: it is expected that they will be concentrated at the beginning and at the end of the SS-CIR. Therefore, a suboptimal solution to the problem, provided that no knowledge of the actual channel paths is available, is given by the Modified NRA, (MNRA), which is formulated as:

$$\begin{aligned} \mathbf{h}_{mnra} &= \mathbf{F}_{um}\mathbf{g}_{mnra} \\ &= \mathbf{F}_{um}(\mathbf{F}_{pm}^H\mathbf{F}_{pm} + \gamma_{mnra}\mathbf{I}_m)^{-1}\mathbf{F}_{pm}^H\mathbf{h}_{ls} \end{aligned} \quad (19)$$

where the matrix $\mathbf{F}_{um} \in \mathbb{C}^{N_u \times N_m}$ is defined as:

$$\mathbf{F}_{um}[k, n] = \begin{cases} \mathbf{F}_u[k, n], & 0 \leq n \leq \lceil N_m(1 - \alpha) \rceil - 1 \\ \mathbf{F}_u[k, N_{fft} - N_m + n], & \lceil N_m(1 - \alpha) \rceil \leq n \leq N_m - 1 \end{cases} \quad (20)$$

and $\mathbf{F}_{pm} \in \mathbb{C}^{N_p \times N_u}$ is defined analogously with respect to \mathbf{F}_p . \mathbf{I}_m is the identity matrix of size N_m . Furthermore, the parameter γ_{mnra} is selected to be $\gamma_{mnra} = N_m\sigma_w^2$, analogously to the sample-spaced case.

Two parameters shall be adapted depending on the PDP and σ_w^2 : N_m representing the number of samples of the equivalent SS-CIR to estimate, and α representing the proportion of the estimated samples in the final part of the SS-CIR.

2) *ENRA and Wiener Filter*: When using the ENRA or the Wiener Filter estimator, it is assumed that the delays of the channel are perfectly known, so that there is no need to estimate the equivalent SS-CIR. Instead, the parameters

to estimate are the complex gains a_i of each of the paths, represented by the vector \mathbf{a} . The estimators can be rewritten for the NSS scenario as:

$$\mathbf{h}_{enra} = \mathbf{T}_u \mathbf{a}_{enra} = \mathbf{T}_u (\mathbf{T}_p^H \mathbf{T}_p + \gamma_{enra} \mathbf{I}_t)^{-1} \mathbf{T}_p^H \mathbf{h}_{ls} \quad (21)$$

$$\mathbf{h}_{wf} = \mathbf{T}_u \mathbf{a}_{wf} = \mathbf{T}_u (\mathbf{T}_p^H \mathbf{T}_p + \sigma_w^2 \mathbf{R}_{aa}^{-1}) \mathbf{T}_p^H \mathbf{h}_{ls} \quad (22)$$

where the matrices $\mathbf{T}_u \in \mathbb{C}^{N_u \times N_t}$ and $\mathbf{T}_p \in \mathbb{C}^{N_p \times N_t}$ are defined with respect to \mathbf{T} in the same way as \mathbf{F}_u and \mathbf{F}_p with respect to \mathbf{F} . As in the SS case, $\gamma_{enra} = \sigma_w^2 N_t$, and $\mathbf{R}_{aa} \in \mathbb{C}^{N_t \times N_t}$ is the correlation matrix of the channel gains, i.e., $\mathbf{R}_{aa} = \text{diag}\{E\{|a_0|^2\}, \dots, E\{|a_{N_t-1}|^2\}\}$ as we assume i.i.d. channel taps. It can be seen that these definitions of the ENRA and WF estimator are equivalent to (10) and (12) when the channel is restricted to be sample-spaced.

3) *MSE of the Estimators*: Unlike the SS scenario, it is difficult to find a general expression that includes all the studied algorithms for an NSS channel. For this reason, we will study the performance of a generic estimator such as:

$$\mathbf{h}_{est} = \mathbf{M}_{est} \mathbf{h}_{ls} \quad (23)$$

which includes any linear estimator that can be expressed in matrix form. With this formulation, the MSE over an NSS channel is:

$$\begin{aligned} \overline{\text{MSE}}\{\mathbf{h}_{est}\} = \frac{1}{N_u} \text{tr} \left\{ \mathbf{T}_u \mathbf{R}_{aa} \mathbf{T}_u^H \right. \\ \left. - \mathbf{T}_u \mathbf{R}_{aa} \mathbf{T}_p^H \mathbf{M}_{est}^H - \mathbf{M}_{est} \mathbf{T}_p \mathbf{R}_{aa} \mathbf{T}_u^H \right. \\ \left. + \mathbf{M}_{est} \mathbf{T}_p \mathbf{R}_{aa} \mathbf{T}_p^H \mathbf{M}_{est}^H + \sigma_w^2 \mathbf{M}_{est} \mathbf{M}_{est}^H \right\} \end{aligned} \quad (24)$$

and the specific values of \mathbf{M}_{est} for each studied algorithm are:

$$\mathbf{M}_{est} = \begin{cases} \mathbf{F}_{um} (\mathbf{F}_{pm}^H \mathbf{F}_{pm} + \gamma_{mnra} \mathbf{I}_m)^{-1} \mathbf{F}_{pm}^H, & \text{MNRA} \\ \mathbf{T}_u (\mathbf{T}_p^H \mathbf{T}_p + \gamma_{enra} \mathbf{I}_t)^{-1} \mathbf{T}_p^H, & \text{ENRA} \\ \mathbf{T}_u (\mathbf{T}_p^H \mathbf{T}_p + \sigma_w^2 \mathbf{R}_{aa}^{-1}) \mathbf{T}_p^H, & \text{WF} \end{cases} \quad (25)$$

IV. PERFORMANCE EVALUATION

In the following, the performance of the estimators discussed in section III will be studied via Monte Carlo simulations. A single-input single-output OFDM system with physical layer parameters proposed for the downlink of UTRA LTE will be used [1]. QPSK modulation is used for both pilot and data symbols. Evenly spaced pilot symbols with a spacing of $\Delta_p = 6$ subcarriers are transmitted in every OFDM block.

Two channel power delay profiles, with 20 equispaced taps and a decay of 1dB per tap leading to an overall loss of 19dB, are used for this simulation study. The “long SS” profile is sample spaced of length $3.711 \mu\text{s}$ and the “long NSS” profile is not sample spaced differing by $0.5 T_s$ added to all delays of the “long SS” profile.

Results for Bit Error Rate (BER) using the studied estimators as a function of the Signal-to-Noise Ratio (SNR) will be given.

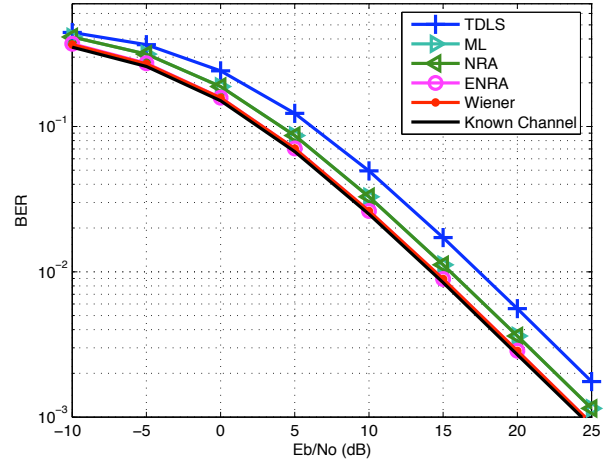


Fig. 2. Performance of the different estimators in a Full Bandwidth OFDM system ($N_u = N_{fft} = 2048$) and a pilot spacing of 6 for the “long SS” channel.

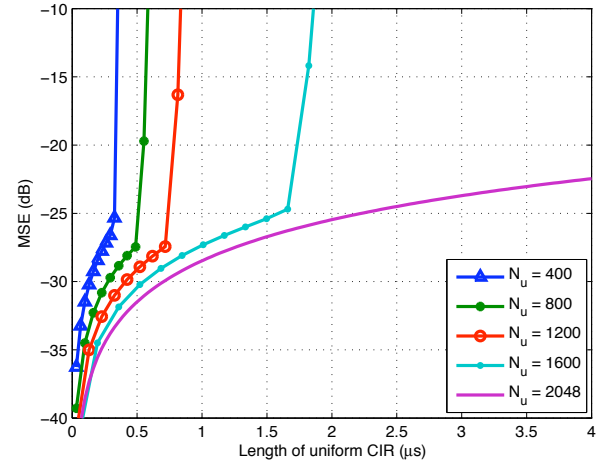


Fig. 3. MSE of the ML estimator for varying assumed CIR length and different N_u , $N_{fft} = 2048$ and $E_b/N_0 = 15$ dB

A. Sample-Spaced Scenario

The performance of the different studied algorithms in a Full Bandwidth system using the “long SS” channel profile is depicted in Fig. 2. From the BER results shown in IV-A, we see that the TDLS curve lies 3.5 dB from the known channel performance at $\frac{E_b}{N_0} = 10$ dB, whereas this distance is reduced to 0.25 dB for the ENRA and WF estimators.

In the case of partial bandwidth the TDLS totally fails, due to bad conditioning. The ML fails as seen on Fig. 3, leading to ill-conditioning of the matrix to be inverted, when the size of the CIR is large for a given N_u .

B. Non-Sample-Spaced Scenario

The effect of having an NSS PDP on the classical algorithms is studied. The BER results are given in Fig. 4 for the NRA, MNRA, ENRA and WF using the “long NSS” channel profile. It is noted that the ENRA and WF have the same performance

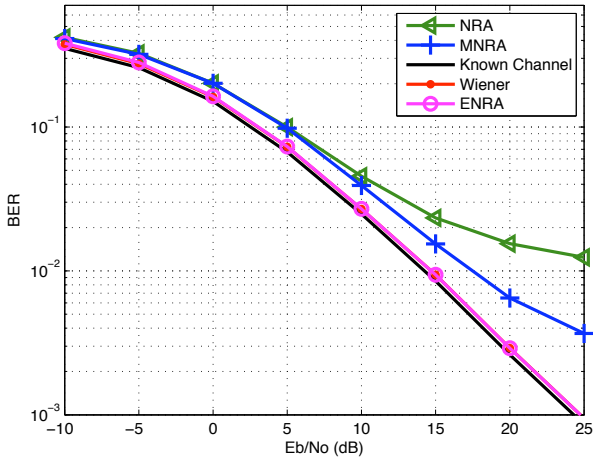


Fig. 4. Effect of leakage on the classical algorithms in an LTE scenario with $N_u = 1200$, $N_{fft} = 2048$ and a pilot spacing of 6 for the “long” NSS channel.

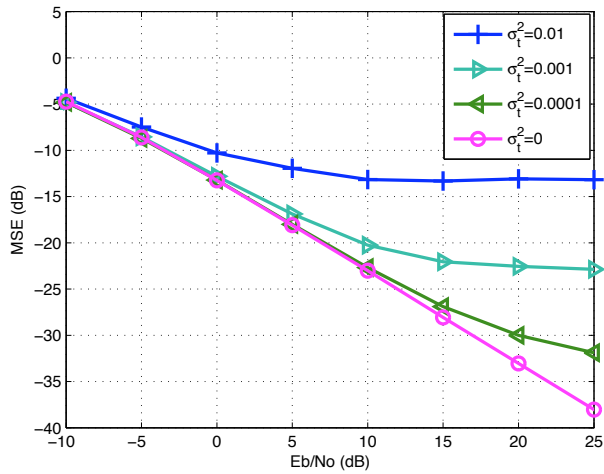


Fig. 5. MSE of the ENRA with different delay estimation errors in an LTE scenario with $N_u = 1200$, $N_{fft} = 2048$ and a pilot spacing of 6 for the “long” NSS channel.

as when employing the “long SS” PDP. The NRA, on the other hand, suffers from significant degradation for $\frac{E_b}{N_0} \geq 10$ dB in both MSE and BER. From these results it can be observed that the knowledge of the tap delays of the PDP is of crucial importance to avoid the leakage effect.

In Fig. 5 the robustness of the ENRA against delay estimation errors is studied. A random zero-mean Gaussian error with variance σ_τ^2 has been added to the delay’s values to simulate imperfect delay estimates, and the MSE of the estimates has been represented. The results show that even with small errors the ENRA suffers from severe degradation as the SNR increases. Very high accuracy in the tap delay estimates is therefore needed in order to avoid leakage.

V. CONCLUSION

In this paper, we have propose a unification of linear PACE OFDM algorithms. Analysis and simulation results are first

given for a sample-spaced channel and Full Bandwidth. The effects of introducing virtual subcarriers as well as a non-sample-spaced channel are studied.

When Partial Bandwidth is used, the TDLS and ML algorithms suffer from severe ill-conditioned matrices and cannot be used as such if the number of virtual subcarriers is too large.

When the channel is non-sample-spaced, exact knowledge of the tap delays is necessary to avoid leakage, with the studied algorithms, but even small errors of tap delay estimates lead to significant performance degradation. This means that without an accurate tap delay estimator the target peak data rates at high SNR in LTE might be compromised.

A modified DFT based robust Wiener seems to be a good candidate for low and middle range SNR (up to 15 dB). However at higher SNR this solution is not recommended and other solutions should be used. These could be based on accurate tap delay estimation or iterative data aided estimation.

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